1 What is Quantitative Finance?

Quantitative finance as a discipline emerged in the 1980s. It is also called financial engineering, financial mathematics, mathematical finance, or, as we call it at Carnegie Mellon, computational finance. It uses the tools of mathematics, statistics, and computer science to solve problems in finance. Computational methods have become an indispensable part of the finance industry. Originally, mathematical modeling played the dominant role in computational finance. Although this continues to be important, in recent years data science and machine learning have become more prominent. Persons working in the finance industry using mathematics, statistics and computer science have come to be known as quants.

Initially relegated to peripheral roles in finance firms, quants have now taken center stage. No longer do traders make decisions based solely on instinct. Top traders rely on sophisticated mathematical models, together with analysis of the current economic and financial landscape, to guide their actions. Instead of sitting in front of monitors “following the market” and making split-second decisions, traders write algorithms that make these split-second decisions for them. Banks are eager to hire “quantitative traders” who know or are prepared to learn this craft.

While trading may be the highest profile activity within financial firms, it is not the only critical function of these firms, nor is it the only place where quants can find intellectually stimulating and rewarding careers. I present below an overview of the finance industry, emphasizing areas in which quantitative skills play a role. Following that I discuss some of the finance...
industry problems whose solutions require quantitative skills. I conclude with observations about quantitative finance at Carnegie Mellon.

2 Overview of the Finance Industry

In a market economy, there is no central planner to determine where capital should be directed to best support production. Instead, that is the role of the financial system.

The finance industry performs the function of capital allocation in a decentralized manner, with many moving parts each making locally optimal decisions. The properties of a well-functioning financial system are the following. First, the capital of savers is put to work by producers. In addition, savers are informed about the risks associated with the allocation of their capital and are rewarded for taking those risks. Producers are provided financial incentives to engage in economic activity. It is possible for all parties to mitigate risk at a reasonable price, or not mitigate risk, as they choose. Finally, the capital extracted from the economy by the financial system itself is small relative to the amount of economic activity facilitated.

These are the objectives of a well-functioning financial system. Governments put in place laws and regulations to create such financial systems. But in addition to laws and regulations, the system needs people with high ethical standards, good communication skills, vision, and increasingly with talent for and knowledge of the mathematical sciences. For many young people, it is their facility with the mathematical sciences that provides entry into the finance industry.

In this section details on the types of financial institutions that require people with quantitative skills are provided. In Section 3 some of the problems these institutions must address are described.

2.1 Sell-Side Firms

Investment banks such as Bank of America Merrill Lynch, Citigroup, Goldman Sachs, J. P. Morgan Chase, and Morgan Stanley create financial products and sell them to institutions, to retail investors, and to one other. For this reason, they are called sell-side firms.

An example of such a financial product is an interest rate swap. Although not as well known to the general public as stocks and bonds, interest rate
swaps play a major role in the allocation of capital within the world-wide economy. An interest rate swap is a contract under which one party agrees to periodically pay a fixed rate of interest accrued on a certain principal, often in the hundreds of millions of dollars, in exchange for receiving a variable rate of interest on the same principal. A corporation that has borrowed money at a variable rate of interest but for planning purposes wishes to have a fixed rate loan can enter an interest rate swap with an investment bank. The effect of the interest rate swap is that the investment bank ends up paying the variable interest rate on the corporation’s loan while the corporation pays a fixed rate of interest to the investment bank.

Another example of a financial product is an exchange rate option. A corporation that plans to build a plant in a foreign country knows that at a future date it will need a large amount of the currency of that country. It does not want to expend its capital now to buy that currency, but it is worried that when the time comes to buy the currency, the exchange rate will have moved in the “wrong direction” and the currency will cost more. On the other hand, the exchange rate might move in the “right direction” and the currency will cost less. To protect itself from the former case while retaining the benefit from the latter case, the firm can buy from an investment bank an exchange rate option that gives it the right at the future date to buy the foreign currency at today’s price or the future price, whichever is less.

Sell-side firms create these and many other products and sell them to clients. At any given time, an investment bank will have hundreds of thousands of ongoing deals on its books. Quants determine the price of these products and how to offset the risk associated with selling them.

2.2 Buy-Side Firms

Not all purchasers of products from sell-side firms are corporations engaged in production. Many are institutional investors, which include pension funds and insurance companies. These entities collect retirement savings or insurance premiums from their customers and must invest this money in financial products that will produce the income that will be needed at the time it will be demanded by their beneficiaries. Others are asset management funds, who receive money from investors, promise to invest it in certain ways, and buy products from sell-side firms to fulfill those promises. Still others are hedge funds, which are taking calculated gambles. Some well-known hedge funds are AQR Capital Management, Citadel, and David Tepper’s Appaloosa.
Management. There are thousands of smaller, less well-known hedge funds. Financial firms that buy assets are collectively called \textit{buy-side firms}.

The topic of hedge funds deserves elaboration. Some hedge funds adopt a view on the markets. Suppose, for example, that a hedge fund becomes convinced that bonds backed by U.S. home mortgages will default. What trade should it make to profit if this is correct? For that, it can go to an investment bank and buy \textit{credit default swaps}, contracts that require the hedge fund to pay a periodic premium but will pay the hedge fund a large sum of money if bonds backed by U.S. home mortgages default. A credit default swap is essentially insurance against default, but it is insurance that a hedge fund can buy even if it does not own the mortgage-backed bond that is being insured. Trades like this took place prior to the financial crisis of 2007–2010, and they figure prominently in the film \textit{The Big Short} about that crisis.

Other hedge funds watch for signals in prices of assets and make trades to profit from what they believe those signals predict. This activity has become especially popular as trading has moved from open outcry to electronic exchanges, so that data from past trades can be collected electronically and analyzed using machine learning methods.

Quants determine which and how many financial instruments to buy in order to implement hedge fund strategies.

### 2.3 Exchanges and Clearing Houses

In 1990 the bid-ask spread for trading on the floor of the New York Stock Exchange (NYSE) was one-eighth of a dollar, and trading was facilitated by \textit{specialists} who would pony up $4 million for a seat on the NYSE. These specialists could then capture the spread on millions of shares traded daily. At the beginning of this century, seats on the NYSE lost 75\% of their value. Today, essentially all trading on the NYSE and elsewhere is done by computers quietly humming along. Instead of specialists having “seats” on the NYSE, firms are now invited to be \textit{market makers}. A market maker must post a price at which it is willing to buy stock and a price at which it is willing to sell. These prices often differ by only one cent, not one-eighth of a dollar. In exchange for this service, the market maker may be provided a small fee for each trade made. Even in the absence of designated market makers, high-frequency traders often play the role of market makers, making milli-second decisions on what prices to post and what prices posted by others to accept.
Exchanges are used for trading stocks, stock options, and many types of futures contracts. For futures positions, exchanges require traders to post cash margins to guarantee that they can fulfill their obligations under the futures contracts they hold.

The move to electronic trading has forced exchanges to develop software, set policies governing trading, and enforce those policies. To monitor trading, exchanges collect and analyze terabytes of data. This falls within the domain of quants.

Other types of contracts, including the interest rate swaps and credit default swaps mentioned above, are not traded on exchanges, but rather are traded over the counter. Two counterparties agree between themselves to enter one of these contracts. However, these counterparties are generally required by regulators to take this contract to a clearing house, where the trade is recorded and the counterparties post cash collateral to ensure they can fulfill their obligations under the contract. Like the exchanges, the clearing house becomes responsible for monitoring performance and enforcing rules. Again, quants are involved.

2.4 Data Providers and Software Vendors

All financial institutions depend on data. There are firms that specialize in providing this data, and in addition to data they generally provide financial software that can be used to analyze the data. A well-known firm that does this is Bloomberg. A smaller but significant competing firm is FactSet. There are also firms that supply software only. One such is Numerix.

Although data providers and software vendors employ many information technology experts, they also need to be knowledgeable about financial markets, the regulatory environment, and the mathematical models behind the software they provide. For this they need quants.

2.5 Regulators

Regulators are charged to make sure financial firms set aside sufficient capital to protect themselves from major losses, and thereby protect the financial system from disruption. To do this, the riskiness of firms’ positions must be measured. The financial firms do this measurement, and regulators monitor the firms to verify that their risk measurement systems are accurate. Therefore, regulators must understand the mathematical models used
to price trades and assess risk. It is no longer the case that the key employees for the regulators are lawyers. Today the leading edge of the regulatory function is carried out by quants.

2.6 Insurance Firms

Insurance firms face a difficult problem. They collect premiums to enable them to cover losses of their beneficiaries, but those losses occur at unknown times and are of unknown size. Between the time the premium is collected and the time the loss occurs, the insurance company invests the premium. In the case of a life insurance policy, this investment may be over multiple decades. There is tremendous uncertainty about the return in financial markets over such a long time period. How should the insurance company invest in order to match its known assets with its uncertain liabilities? What kinds of assets, either that presently exist or that it might ask an investment bank to create, should it hold? Because of the huge role played by financial markets in this asset/liability matching problem, this is more than an actuarial science problem. It is a problem that requires the expertise of quants.

2.7 Rating and Consulting

Ratings agencies such as Moody’s and Standard & Poor’s use mathematical models to assess the likelihood that a firm will default on its debt. For this they hire quants.

One of the functions of consultants such as Ernst & Young is to assist firms who do not have a fully developed quant team in house to develop and/or evaluate the mathematical models they use to govern their interactions with financial markets. In other words, some firms choose to “rent” the quant team of a consulting firm rather than build their own team.

2.8 Fintech

The most recent entry into the array of financial firms needing the expertise of quants are the fintech firms. Many of these are start-up firms who attempt to leverage the availability of data and advances in computational power to disrupt conventional financial processes. One of the more successful business models along these lines is peer-to-peer lending. To compete with these firms, investment banks are also investing heavily in technology. The skills
required to participate in the start-up ventures are business acumen, a strong computer science background, and the ability to handle and analyze large data sets.

3 Quants and Mathematical Models

A mathematical model is a set of idealized assumptions about an object of study that permits one to draw meaningful conclusions and, in particular, obtain quantitative as well as qualitative results. Real world phenomena are complicated and it is necessary to make assumptions that are only approximately correct in order to build a mathematical model. To do this in a way that captures the essence of the phenomenon and leads to a manageable model requires a high level of skill and experience. Not every quant needs to create new models, but a quant needs to understand the mathematical models she or he is using, and that means knowing their limitations.

3.1 The Black-Scholes Model

The most successful mathematical model in finance is due to Fischer Black, Myron Scholes and Robert Merton, and led to the publication in 1973 of the Black-Scholes option pricing formula. For this work, Scholes and Merton won the 1997 Nobel Prize in Economics. (The prize is only awarded to living people, and Black died prematurely.)

The Black-Scholes formula tells how to compute the price of an option to buy a share of stock at a future date at a price agreed upon before that date. While such options are important in their own right, the more important aspect of the work of Black, Merton and Scholes was that it initiated a whole body of research on how to price derivative securities, securities whose price is derived from the price of some other underlying security.

The Black-Scholes formula says the price of an option to buy a share of stock at a price $K$ at a time $T$ units in the future is

$$SN(d_1) - e^{-rT}KN(d_2),$$

where

$$d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \log \frac{S}{K} + \left( r + \frac{1}{2} \sigma^2 \right) T \right], \quad d_2 = d_1 - \sigma \sqrt{T}.$$
In this formula, \( N \) is the standard cumulative normal distribution function (the integral of the bell-shaped curve), given by

\[
N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-x^2/2} dx,
\]

\( S \) is the stock price at the time the option is priced, \( r \) is the interest rate at which money can be borrowed or invested, and the critical parameter \( \sigma \) is the volatility of the stock, a measure of how risky the stock is.

This formula is based on a model in which it is assumed that any number of shares of the stock can be bought or sold at the same price (zero bid-ask spread and zero price impact from trading), there is a constant interest rate at which money can be borrowed or invested during the lifetime of the option, and the volatility of the stock on which the option is written is constant over the lifetime of the option. The derivation of the formula requires a good understanding of calculus and probability and the combination of the two into the theory of stochastic calculus. Its use as a guide for buying and selling options requires one to also understand financial markets. The invention of the Black-Scholes formula enabled extensive growth in options trading. It also sparked a largely successful search for similar formulas for more complicated derivative securities and for models in which the Black-Scholes assumptions do not hold. This revolutionized the finance industry.

stocks belong to equity markets because they derive their value from the equity (value) of the issuing firm. Other securities, whose value is primarily determined by interest rates, are said to be part of fixed-income markets. Currencies and commodities are other asset classes traded in financial markets. These other asset classes, sometimes collectively called FICC for fixed income, currencies and commodities, provide underlying securities for derivative securities. Such derivative securities, which include the interest rate swaps and credit default swaps discussed earlier, are priced and traded using models that have grown out of the Black-Scholes framework. Understanding the fundamental ideas behind these models is the first step in becoming a quant.

### 3.2 Mortgage-Backed Securities

We single out for discussion a particularly large and important asset class. Since the 1980s there has been a fundamental change in the way the U.S.
mortgage market is financed. Prior to that time, individual investors deposited money with banks, particularly with a type of bank called a Savings and Loan. These banks would in turn lend their deposits in the form of mortgages to persons buying real estate.

But many investors, particularly those with large net worth, sought investment opportunities other than those provided by Savings and Loans, and their money was therefore unavailable to persons wanting mortgages. In the 1980s this changed as major investment banks and the quasi-governmental agencies Fannie Mae and Freddie Mac began buying mortgages from Savings and Loans and other mortgage originators and selling off pieces of the pool of mortgages they collected to investors in the form of mortgage-backed bonds. These bonds were structured to appeal to the risk/return appetites of individual investors.

Structuring mortgage-backed bonds is a complex problem because borrowers may default, and in the U.S., they have the option without penalty to repay the principal partially or fully ahead of schedule. Lenders must be compensated commensurately with the default risk to which they are exposed. In addition, some lenders want to be repaid quickly, whereas others prefer to lend their money for a long period of time. To match borrowers to lenders, mortgage-backed bonds are based not on the income from a single mortgage but on the income from a collection of hundreds of mortgages. Such a pool of mortgages receives scheduled principal and interest payments, incurs some defaults, and also receives some principal payments ahead of schedule. Mortgage-backed bonds are structured so that different bonds are affected by these random events in different ways, thereby making the different bonds attractive to different kinds of investors. The determination of the prices of these bonds is much more than an accounting exercise. One needs to know statistics in order to estimate how many mortgages will default and how many will pay off principal ahead of schedule, one needs a good sense of finance and mathematics to model the effect of changes in the interest rate on pre-payments and the value of the mortgages, and one needs the ability to write efficient computer programs to perform the necessary large-scale computations to actually get numerical results.

In the case of mortgage-backed bonds, the common mathematical model for pricing, the so-called Gaussian copula model, was applied in situations for which it was wholly inappropriate. This fact was either not appreciated or willfully ignored by both buyers and sellers of mortgage-backed bonds, and that ultimately led to a major financial crisis. The best reason to learn the
mathematical models behind pricing and trading is to be able to determine when the models are being stretched beyond their limits. This point was emphatically demonstrated during the collapse of the U.S. housing market beginning in 2007.

3.3 Asset Management

Most people approach financial markets with an eye toward making good investments rather than pricing derivative securities. Mathematics entered this branch of finance with the work of Harry Markowitz in 1952, which laid the groundwork for a mathematical model to determine the risk and expected return associated with a portfolio of stocks. Perhaps because the mathematics of Markowitz’s mean-variance analysis was simple compared to what is encountered in derivative securities pricing, Markowitz’s ideas were adopted quickly by asset managers and became standard fare in MBA programs. Together with Merton Miller and William Sharpe, Markowitz won the 1990 Nobel Prize in Economics. From the time Markowitz’s ideas first gained acceptance until about the time the Nobel Prize was awarded, there was little change in the methods used for asset management.

By contrast, in recent years we are seeing firms who manage assets adopt much more sophisticated mathematical methods. One reason for this is that in addition to stocks and bonds, there are now a whole host of derivative securities that can be used for investment, but in order to invest wisely in these securities, one must understand them. Another reason seems to be that many people who began their careers pricing and trading derivative securities subsequently moved into asset management, where they applied the same level of quantitative expertise as when they were on the sell side. This is certainly the case in hedge funds, but even more conservative asset managers are building models, developing algorithms to guide their trading, and conducting statistical back-tests on these algorithms. This work is being carried out by quants.

3.4 Optimal Execution

If someone is trying to buy many shares of an asset, submitting that order to an exchange will cause the price to move up. It is a mistake to buy a large block of shares all at once because this will “run the book;” all the shares available at a certain price will be bought, then the shares at the next
higher price, and so on until the full block of shares has been bought. In this case, only the first few shares are bought at the price available in the market at the time the trade was initiated. It is better to submit only part of the block to the exchange and wait for the impact of that submission to subside before submitting another part of the block. However, it takes more time to buy a large block when submitting it piecemeal, and during that time the price available on the exchange may move up for reasons unrelated to the submission of this block.

In recent years mathematical models have been developed to describe the impact placing an order on an exchange has on the price, and within these models one can optimally balance the trade-off between price impact from order submission and the price change risk incurred by trading slowly. The practice of balancing these competing objectives is called \textit{optimal execution}. Firms such as Quantitative Brokers and groups within investment banks offer optimal execution services to clients. The optimal execution algorithms behind these services are developed and maintained by quants.

\section*{3.5 Risk Management}

The risk management function at systemically important financial institutions has changed dramatically since the financial crisis of 2008–2010. It is no longer the case that the recommendations of the risk management team within an investment bank can be overruled by a successful trader. Today the Chief Risk Officer of a bank sits in the executive suite. She or he has direct access to the Chief Executive Officer and she or he presents the risk profile of the firm to the Board of Directors.

The Chief Risk Officer is supported by computations of the credit and market risk borne by trading operations and portfolio management that have increased in complexity and importance since the financial crisis. As one example, consider the determination of the credit valuation adjustment (CVA) mandated by the Dodd-Frank Act. For each deal in the bank’s portfolio, the CVA takes into account the risk of counterparty default and the potential loss in the event of such default. This must be done for the whole portfolio of the bank, which can contain hundreds of thousands of long-lived deals. The CVA must be computed on a regular basis, and the bank is required to set aside risk capital that depends on the degree of CVA fluctuations observed over time. Some banks enter trades to dampen these fluctuations, and determining the nature and size of such trades is a major challenge. All these
activities fall within the purview of quants.

4 Computational Finance at Carnegie-Mellon

In 1994 Carnegie Mellon created MSCF (Master of Science in Computational Finance), the first professional degree in quantitative finance. MSCF graduates 90 to 100 students annually. Over the years the alumni network has become quite large. Some alumni have risen to influential positions in the finance industry. Alumni advise faculty, return to campus to meet students, conduct mock interviews for students, and generally make themselves available. They are a valuable asset of MSCF.

Another valuable asset is the faculty. Quantitative finance professionals need skills in multiple disciplines, and MSCF has faculty in all these disciplines. Mathematical models describe observed phenomena, but the models are never completely correct and knowing when to trust and when not to trust the output of such models requires a good understanding of both the mathematics involved and the phenomenon being studied. This is the reason MSCF includes both mathematics and finance. As terabytes of financial data have become readily available, data science and statistics are playing a more important role in quantitative finance. MSCF has an embedded data science program. Finally, every quant needs to know how to write computer code. Programming instruction is part of MSCF.

All of this is possible because MSCF is a partnership of four departments in four different colleges. These are the Tepper School of Business, the Department of Statistics and Data Science, the Department of Mathematical Sciences, and the Heinz College of Information Systems and Public Policy. MSCF is the only quantitative finance program with faculty drawn from such a wide range of disciplines. MSCF courses are designed and taught by faculty in these four departments, sometimes in collaboration with industry practitioners, so that the program covers the range of skills needed by modern quantitative finance professionals. The participation of faculty from multiple departments enables MSCF to adapt the curriculum as the needs of the finance industry evolve.

Finally, MSCF at Carnegie Mellon is part of a deep ongoing university commitment to quantitative finance. The university also offers a Bachelor of Science in Computational Finance, graduates PhD students who enter the finance industry, and conducts research in quantitative methods applied
to finance. MSCF benefits from having the PhD students as teaching assistants and from the reputational advantage conferred on Carnegie-Mellon University by its overall stature in quantitative finance.

5 Preparing for Computational Finance

There are many different types of jobs in quantitative finance. A person seeking to enter the field should explore the possibilities. The person should also take a self inventory of likes and dislikes, assessing what she or he is good at doing, or can become good at doing, and what she or he is not good at doing. A successful career begins with matching a person to a job, and to find a right match a person needs to know herself or himself.

There is more to quantitative finance than one can learn in classes. In his autobiography My Life as a Quant, Emanuel Derman, the physicist who became a partner at Goldman Sachs, writes about his time as an undergraduate. Derman says “I’m still embarrassed to admit to myself that I almost never studied anything I wasn’t officially taught.” Computational finance attracts highly motivated people who study things they are not officially taught, and Derman’s example notwithstanding, doing so can confer a competitive edge. A good place to begin is Derman’s book, Chapters 9 to 15, which describe his years at Goldman Sachs. There are numerous other informative books about all aspects of quantitative finance.

At Carnegie Mellon there are many opportunities to learn things that are not officially taught. One way is from fellow students, some of whom have worked in the finance industry. Another is by meeting the alumni and industry professionals who visit campus. A third is by reading the Wall Street Journal and relevant websites.

People who are successful in computational finance pay attention to current events in the field. In the busy life of a student, it is not possible to read the Wall Street Journal every day, but it is possible to check it a few times each week in order to find one good story about finance and read it carefully. It is better to read a few things in detail than to skim many things and thereby acquire only a shallow understanding. When firms interview for internships and full-time jobs, they tend to first figure out what a student should know or claims to know, and then ask a series of questions, each one probing more deeply than the previous one. A student who claims to know something should be prepared for questioning about it.
6 Conclusion

Society needs talented, well-trained quantitative finance professionals. The variety of firms that hire quants is great, and there is a wide range of problems whose solutions require the skills of quants. A career in quantitative finance offers the opportunity to work alongside smart people on challenging, important tasks that use mathematics, statistics, finance, and computer science.
Positions in finance are intellectually challenging, critical to the economy and well compensated. While it is possible to enter the finance industry with a STEM (Science, Technology, Engineering and Mathematics) education at any level, many choose to enter after earning a master's degree in quantitative finance.

www.QuantNet.com is an excellent resource to learn more about the field of quantitative finance. Additionally, QuantNet describes thirty quantitative finance masters programs throughout North America. Admission to these programs will generally require a solid background in:

• Multi-variable calculus
• Linear algebra
• Calculus-based probability
• Statistical inference and data analysis
• Programming experience in an object-oriented language (e.g., C++ or Python)
• Good written and oral communication skills
• Ability to work as part of a team
• An interest in the financial markets

Typical coursework in these programs includes stochastic calculus, term structure and linear models, optimization, programming, econometrics, time series, simulation and corporate finance.

To compete globally, our economy needs an efficient and competitive financial system. The health of this system depends upon the talent of people like you – well-educated professionals with a background in mathematics, statistics and computer science – working in the field of quantitative finance.

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